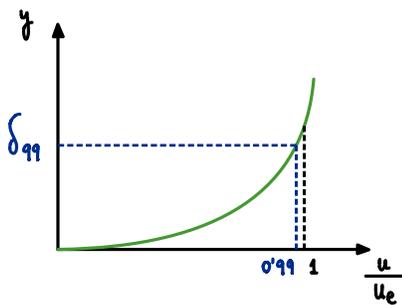


Capa Límite Laminar : Espesores



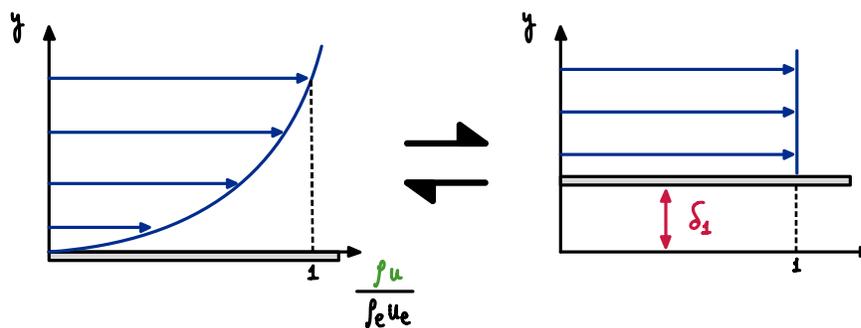
$$\frac{\delta}{l} \sim Re^{-1/2} \rightarrow \text{¿Cómo evaluamos } \delta?$$

$$\delta_{99} = y \Big|_{\frac{u}{u_e} = 0.99}$$

Errores grandes a partir de solución numérica o datos experimentales

ESPESOR DE DESPLAZAMIENTO δ_1

DISTANCIA QUE HABRÍA QUE LEVANTAR LA PARED PARA QUE EL GASTO DE FLUIDO A LA VELOCIDAD EXTERIOR CONSTANTE (COMO SI NO HUBIERA CAPA LÍMITE) COINCIDA CON EL DEL FLUIDO DE LA CAPA LÍMITE.



ρu : flujo másico a través de la capa límite

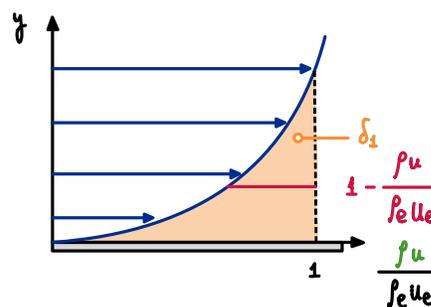
$$\text{Déficit de flujo másico: } \rho_e u_e - \rho u = \rho_e u_e \left(1 - \frac{\rho u}{\rho_e u_e}\right)$$

Matemáticamente :

$$\int_0^{\infty} \rho u dy = \int_{\delta_1}^{\infty} \rho_e u_e dy = \int_0^{\infty} \rho_e u_e dy - \int_0^{\delta_1} \rho_e u_e dy = \int_0^{\infty} \rho_e u_e dy - \rho_e u_e \delta_1$$

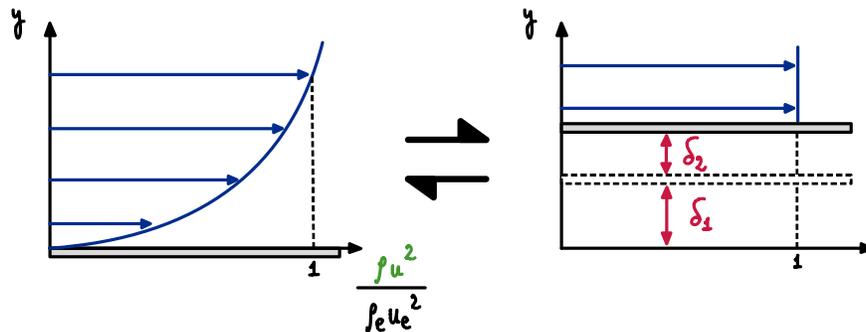


$$\delta_1 = \int_0^{\infty} \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy$$



ESPESOR DE CANTIDAD DE MOVIMIENTO δ_2

DISTANCIA QUE HABRÍA QUE LEVANTAR LA PARED SOBRE δ_1 PARA QUE EL FLUJO DE CANTIDAD DE MOVIMIENTO DEL FLUIDO A LA VELOCIDAD EXTERIOR CONSTANTE (COMO SI NO HUBIERA CAPA LÍMITE) COINCIDA CON EL DEL FLUIDO DE LA CAPA LÍMITE.

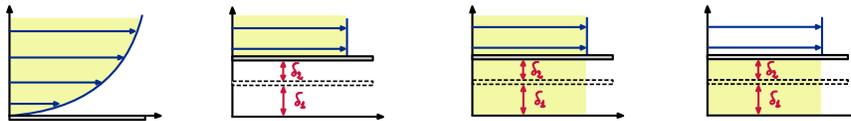


ρu^2 : flujo de cantidad de movimiento a través de la capa límite

Déficit de flujo de cantidad de movimiento: $\rho_e u_e^2 - \rho u^2 = \rho_e u_e^2 \left(1 - \frac{\rho u^2}{\rho_e u_e^2} \right)$

Matemáticamente:

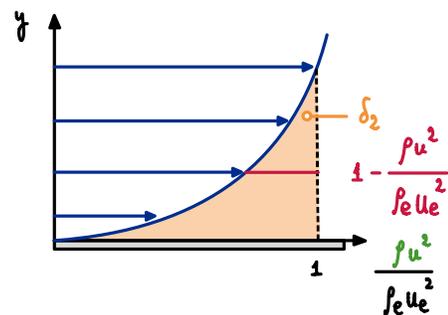
$$\int_0^{\infty} \rho u^2 dy = \int_{\delta_1 + \delta_2}^{\infty} \rho_e u_e^2 dy = \int_0^{\infty} \rho_e u_e^2 dy - \int_0^{\delta_1} \rho_e u_e^2 dy = \int_0^{\infty} \rho_e u_e dy - \rho_e u_e^2 (\delta_1 + \delta_2)$$



$$\delta_2 = \frac{1}{\rho_e u_e} \int_0^{\infty} (\rho_e u_e^2 - \rho u^2) dy - \delta_1 = \int_0^{\infty} \left(1 - \frac{\rho u^2}{\rho_e u_e^2} \right) dy - \int_0^{\infty} \left(1 - \frac{\rho u}{\rho_e u_e} \right) dy$$

$$\delta_2 = \int_0^{\infty} \frac{\rho u}{\rho_e u_e} \left(1 - \frac{u}{u_e} \right) dy$$

Como $u < u_e \rightarrow \delta_2 < \delta_1$

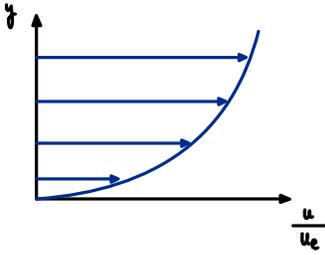


FACTOR DE FORMA H_{12}

$$H_{12} = \frac{\delta_1}{\delta_2}$$

Como $\delta_2 < \delta_1 \rightarrow H_{12} > 1$

En una placa plana sometida a flujo incompresible con $\nabla p_e = 0$:

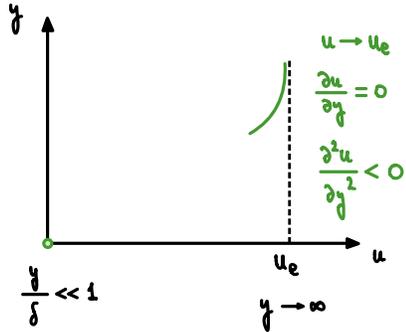


$$\frac{\delta_1}{\delta_{99}} \sim \frac{1}{3} \quad ; \quad \frac{\delta_2}{\delta_{99}} \sim \frac{2}{5} \quad \rightarrow \quad H_{12} \sim 2.5$$

Succión/Soplado: POROSIDAD $\begin{cases} v_p > 0 & \text{SOPLADO} \\ v_p < 0 & \text{SUCCIÓN} \end{cases}$

¿Qué pasa muy cerca de la pared? \rightarrow Estación x

Desarrollo de Taylor para u:



$$u = u|_{y=0} + \frac{\partial u}{\partial y}|_{y=0} y + \frac{\partial^2 u}{\partial y^2}|_{y=0} \frac{1}{2} y^2 + o(y^3)$$

$$u|_{y=0} \rightarrow u|_{y=0} = 0$$

$$\frac{\partial u}{\partial y}|_{y=0}$$

$$\begin{aligned} \overline{\tau}_\mu &= 2\mu \overline{S} \\ \overline{\tau}_\mu|_{y=0} &= 2\mu \overline{S}|_{y=0} = 2\mu \left[\frac{\partial u}{\partial x}|_{y=0} + \frac{\partial v}{\partial x}|_{y=0} \right] \end{aligned}$$

$\sim \frac{u_e}{\delta}$ $\sim \frac{u_e}{\ell} \sim u_e \frac{\delta}{\ell^2}$

$$\frac{1}{2} \left(\frac{\partial u}{\partial y}|_{y=0} + \frac{\partial v}{\partial x}|_{y=0} \right) \approx \mu \begin{bmatrix} 0 & \frac{\partial u}{\partial y}|_{y=0} \\ \frac{\partial u}{\partial y}|_{y=0} & 0 \end{bmatrix}$$

$$\vec{n}_p = \vec{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

VOLUMEN DE CONTROL

$$\vec{\tau}_p = \overline{\tau}_\mu|_{y=0} \cdot \vec{n}_p = \mu \begin{bmatrix} 0 & \frac{\partial u}{\partial y}|_{y=0} \\ \frac{\partial u}{\partial y}|_{y=0} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \mu \frac{\partial u}{\partial y}|_{y=0} \vec{i} \rightarrow \frac{\partial u}{\partial y}|_{y=0} = \frac{\tau_p}{\mu}$$

$$\frac{\partial^2 u}{\partial y^2}|_{y=0}$$

Recurrimos a la ECDM_x|_{y=0}:

$$\underbrace{u \frac{\partial u}{\partial x}}_{0}|_{y=0} + \underbrace{v \frac{\partial v}{\partial y}}_{v_p \frac{\tau_p}{\mu}}|_{y=0} = u_e \frac{du_e}{dx}|_{y=0} + \nu \frac{\partial^2 u}{\partial y^2}|_{y=0} \rightarrow \frac{\partial^2 u}{\partial y^2}|_{y=0} = \frac{1}{\nu} \left(v_p \frac{\tau_p}{\mu} - u_e \frac{du_e}{dx} \right)$$

Por tanto:

$$u \approx \frac{\tau_p}{\mu} y + \frac{1}{\nu} \left(-u_e \frac{du_e}{dx} + v_p \frac{\tau_p}{\mu} \right) \frac{y^2}{2}$$

Desarrollo de Taylor para v :

$$v = v|_{y=0} + \frac{\partial v}{\partial y}|_{y=0} y + \frac{\partial^2 v}{\partial y^2}|_{y=0} \frac{1}{2} y^2 + o(y^3)$$

$$v|_{y=0} \longrightarrow v|_{y=0} = v_p$$

$$\frac{\partial v}{\partial y}|_{y=0}$$

$$\frac{\partial v}{\partial y}|_{y=0} = -\frac{\partial u}{\partial x}|_{y=0} = 0 \quad (u|_{y=0} = 0) \longrightarrow \frac{\partial v}{\partial y}|_{y=0} = 0$$

$$\frac{\partial^2 v}{\partial y^2}|_{y=0}$$

CONTINUIDAD

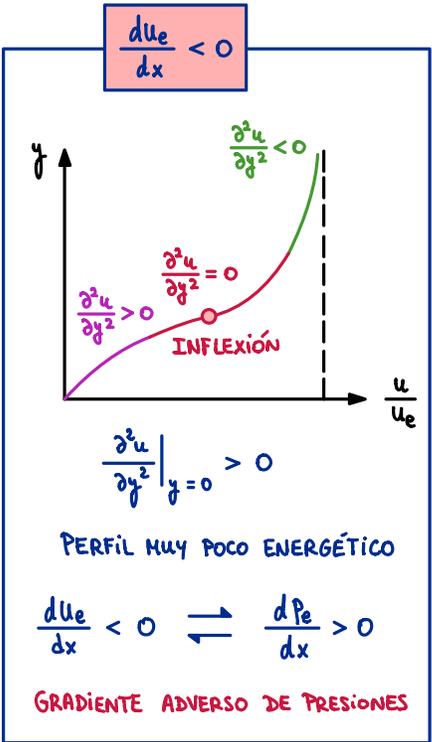
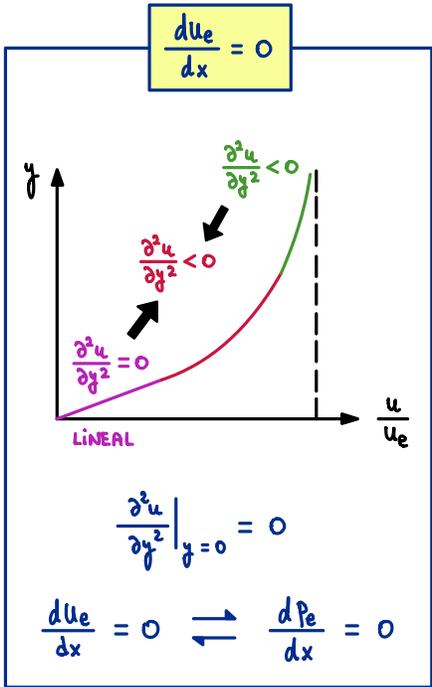
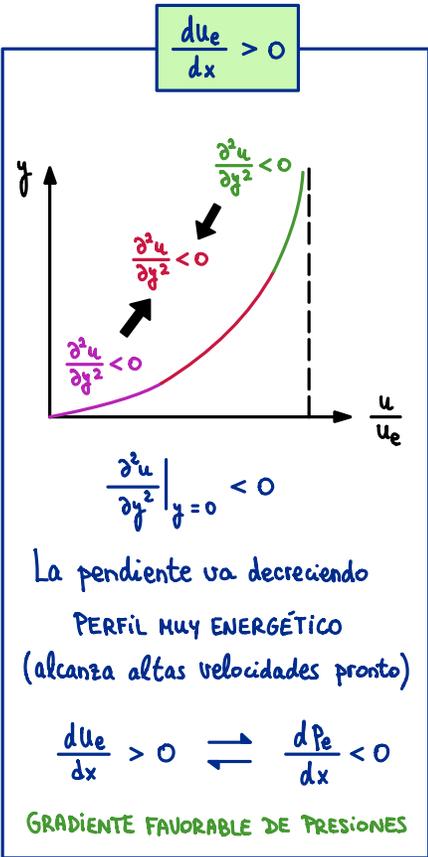
$$\frac{\partial^2 v}{\partial y^2}|_{y=0} = -\frac{\partial^2 u}{\partial x \partial y}|_{y=0} = -\frac{\partial}{\partial x} \left(\frac{\partial u}{\partial y}|_{y=0} \right) = -\frac{d}{dx} \left(\frac{v_p}{\mu} \right) = -\frac{1}{\mu} \frac{d v_p}{dx} \longrightarrow$$

Por tanto :

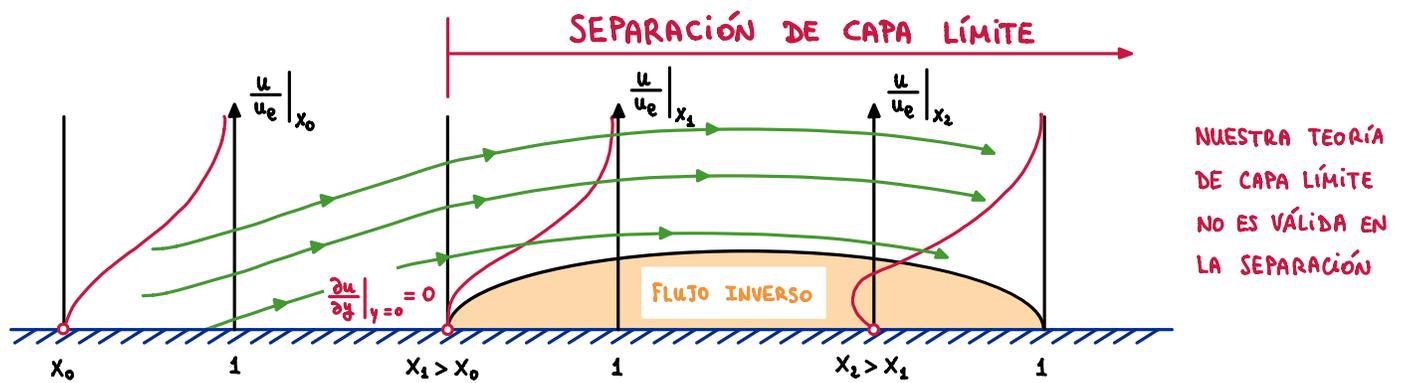
$$\frac{\partial^2 v}{\partial y^2}|_{y=0} = -\frac{1}{\mu} \frac{d v_p}{dx}$$

$$v \approx v_p - \frac{1}{2\mu} \frac{d v_p}{dx} y^2$$

Para $v_p = 0$:



Si el gradiente adverso de presiones persiste :



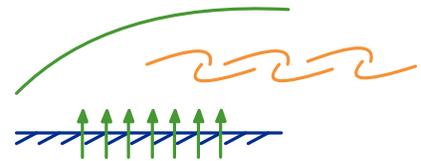
La separación de la capa límite se produce por :

$$\frac{du_e}{dx} < 0$$

GRADIENTE ADVERSO

$$U_p > 0$$

SOPLADO



Podemos succionar la capa límite para contrarrestar un gradiente adverso de presiones :

$$\text{PUNTO DE SEPARACIÓN : } \frac{\partial u}{\partial y} \Big|_{y=0} = 0 \iff C_p = 0$$

